

# Modelling and Forecasting Environmental Protection Expenditures: A Comparative Study of Alternative Methods

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# Abstract

Forecasting environmental protection expenditures is important in planning the financing resources needed for policies aimed at environmental protection. In this study, we aimed to determine the highest performing models from the Exponential Smoothing, Box-Jenkins" and Artificial Neural Networks methods and to estimate the environmental protection expenditures of metropolitan municipalities, which have a major role in the realization of environmental protection activities in Türkiye, for the periods from 2024:-2025: 4. To evaluate the performance of the models, we used the Mean Absolute Percentage Error (MAPE) criterion and identified the model with the lowest MAPE as the most accurate for forecasting. As a result of the experiments, it was seen that all three models (Seasonal ARIMA, Multiplicative-Seasonal Holt-Winter's and ANN) produced quite successful results, and the Nonlinear Autoregressive ANN model was more successful in capturing the nonlinear patterns in the data compared to the time series models, albeit with a small difference.

**Keywords:** Environmental Protection Expenditures Forecasting, Exponential Smoothing, Box-Jenkins (ARIMA), Artificial Neural Networks

# 1. INTRODUCTION

Environmental protection expenditure is the cost of measures taken to prevent, minimise and eliminate environmental damage caused by the production and consumption of products and services (Broniewicz, 2001). The Turkish Statistical Institute has categorised environmental protection expenditures into nine different areas, following the framework of the UN Statistical Commission's Classification of Environmental Protection Activities (CEPA). These categories include air and climate protection, wastewater management, waste management, soil, groundwater and surface water protection and remediation, noise and vibration control, biodiversity and landscape protection, radiation protection, research and development activities, and other environmental protection initiatives (TÜİK, 2024). Local governments are at the forefront of efforts to protect the environment by eliminating environmental problems at their source, as environmental problems arise first at the local level and have the potential to spread beyond local boundaries to a large area on a global scale. Local governments were given the responsibility to develop and implement the Agenda 21 document for the provision of sustainable development at the national level, which was enacted after the Rio Summit in 1992 (Çitfçioğlu and Aydın, 2019: 118). In Turkey, local government organizations are responsible for 80 percent of the costs associated with environmental protection. The fact that local governments have a legal obligation to protect the environment is one of the most important factors

contributing to this situation (HVMB, 2021: 29). It is widely recognized that environmental problems are a consequence of population growth (Chiras, 2013). In this regard, it can be stated that the expectations for the activities and projects undertaken by metropolitan municipalities, which host a significant proportion of the population of Turkey, are high (TÜİK, 2022).

Estimates of the primary causes that lead to expenditures are commonly included in Budget and Public Finance Research, whereas forecasts of expenditures are less common. This lack of expense forecasting may constitute a risk in budgeting (Williams and Calabrese, 2016: 132). The lack of studies in the literature on the estimation of the environmental protection expenditures of metropolitan municipalities in Türkiye using the conventional models used in the time series or the artificial neural network (ANN) method for the future periods was one of the primary drivers for the realization of this study. The majority of studies in the literature are concerned with projecting governmental spending, revenues, budget deficits, and budget balance. Most of the studies in question are based on causal models included in quantitative forecasting models and do not include forecasts for the future. The following lines summarize these studies. Hansen and Nelson (1997) employed Exponential Smoothing and ARIMA methods applied to ANN and time series to estimate the income of the state of Utah in the USA, which is dependent on consumption taxes, as well as the growth in non-farm employment. Additionally, they contrasted the ANN approach with other econometric approaches. According to the findings of the study, all three forecasting approaches yielded results indicating that consumption tax revenues will increase, and when compared to the data announced for 1997, the ANN and ARIMA models produced exceptionally successful outcomes. According to the findings of the study, traditional approaches performed better in short-term predictions, whereas the ANN method performed better in longterm predictions. Yusof (2005), investigated the effect of the number of neurons, activation functions used, and data pre-processing techniques in the design of the ANN forecast model on the performance of the back propagation (BP) network in time series revenue estimation using Malaysian Customs Administration revenues. According to the empirical results, an ANN model with fewer neurons and a more compact network structure yields predictions that are more accurate, and the performance of an ANN model's predictions can be greatly enhanced by using the right pre-processing method. Additionally, it was mentioned that the sigmoid activation function has the fastest convergence and generally has strong prediction ability, reducing the complexity of ANN. Mao et al. (2010) created a model that predicts China's regional general budget revenues by dynamically combining an explanatory BP (back propagation) neural network with a time series BP neural network. In the study, factors that have an impact on regional financial income (Value added tax, business tax, personal income tax, urban maintenance and construction tax, property tax, stamp duty, fines, administrative fees, wage income, deficit subsidies, and government operating income) were investigated using the explanatory BP neural network. According to findings, while the explanatory neural network and the time series neural network model accurately predict budget revenues, the dynamic combination model that incorporates both produces more effective results. In their study, Hotunoğlu et al. (2013) investigated the viability of using ANN to gauge Türkiye's budget balance. They modelled budget deficits as a function of unemployment, inflation, public debt, general elections, local elections, and economic crises using annual data from 1980 to 2010. Türkiye's budget balance for the years 2004 to 2010 was approximated using ANNs. They concluded that the ANN approach, which is very near to the estimate's findings and the actual budget balance, could be utilized to estimate the Turkish budget balance. Magdelena et al. (2015) used feed forward ANN to develop a forecasting model to analyse the relationship of key public expenditures with real GDP growth in selected Central and Eastern European countries in their study from 2015 to 2016. The study covered basic public expenditures such as social protection, health, education, and public services, and distinct models were developed for each. The GDP

ratio was calculated using annual data for the years 2000 through 2013. The model considers the variables that determine the kind of spending for each public expenditure. Konu and Ata (2022) employed "Adaptive Neuro-Fuzzy Inference System" (ANFIS) method to estimate Türkiye's 2014-2019 budget deficit. They addressed the budget deficits using Budget Deficit/GDP ratios, treating the deficits as the dependent variable. Additionally, they calculated the budget deficits using the least squares (LCC) method to compare the results and found that the suggested NFS model produced more insightful outcomes than the LCC model.

Scientifically grounded quantitative forecasts are essential for fiscal policy, allowing timely interventions before fiscal imbalances become unmanageable (Leal et al., 2008). Accurate identification of environmental protection expenditures is crucial for ensuring adequate financial resources for policy implementation. Türkiye's commitments to international environmental agreements largely rely on metropolitan municipalities' action plans, which often require significant investments in sustainable technologies. Therefore, precise estimates of government spending on environmental protection are vital for effective long-term planning. This study aims to identify the best-performing models among Exponential Smoothing, Box-Jenkins, and Artificial Neural Networks to forecast environmental protection expenditures by metropolitan municipalities for the periods 2024:1 – 2025:4. The following sections of the study are structured as follows: In the Data and Method section, we describe the data used in the research and the methodologies applied, including Exponential Smoothing, Box-Jenkins (ARIMA), and Artificial Neural Networks (ANN). The Empirical Findings section presents the results of the forecasting models, comparing the performances of the different approaches. Finally, the Conclusion section provides a discussion of the implications of the findings, along with limitations and directions for future research.

## 2. DATA AND METHOD

In this study, we used the environmental protection expenditure data of metropolitan municipalities in Türkiye for the period 2007:1 to 2024:2. We obtained the data from the general government financial statistics available on the official website of the Directorate General of Accounting Department of the Ministry of Treasury and Finance of the Republic of Türkiye. The data presented as quarterly cumulative totals that reset each year. We examined the time series components of the data to determine suitable methods based on the structure of the data used in the research. We estimated the environmental protection expenditures of metropolitan municipalities for the 2007:1 to 2024:2 period using Exponential Smoothing and Box-Jenkins (ARIMA) methods, along with Artificial Neural Network (ANN) models with different architectural structures. The best model among the used model was found after the estimation results produced by the models were compared with the actual values of the environmental protection expenditures using the selected of model accuracy measures. For any forecasting technique, it is important to measure the accuracy of its forecasts. In the literature, statistics such as Mean Square Error (MSE), "Mean Square Error (RMSE), Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE)" can be used to measure the prediction performance of models. The MAPE statistic allows for easier understanding of estimation errors since it gives proportional results, that minus and plus-signed observation values do not affect each other since it is calculated by absolute value, that the observation values are not affected by their size, and that they have a meaning on their own. For this reason, it is frequently preferred in the literature to compare the estimation performances of models (Çuhadar, 2013; Özkan and Aladağ. 2020; Sarı and Gül, 2022). We used MAPE statistic to compare the forecast performances of the methods used in the study. As a result of the evaluations, metropolitan municipalities' environmental protection expenditures were forecasted for the periods 2024:3-2025:4 by using the models with the highest prediction performance among the various methods tried

#### 2.1. Analysis of the Time Series Features of the Data

The time series of environmental protection expenditures of the metropolitan municipalities used in the study for the period 2007:1–2024:2 is shown in the Figure 1. In this figure, the main components affecting the series can be visually described. It can be observed that the time series has an increasing trend and seasonal component with irregular fluctuations in some periods. It is observed that it started to increase from the 2nd quarter and reached the highest value in the 4th quarter and reached the lowest value in the 1st quarter for the next year. When the time series of the data is examined, it is noteworthy that the environmental protection expenditures of Metropolitan Municipalities show a significant increase from the 2nd quarter of 2022 to the 2nd quarter of 2024.



Figure 1. Metropolitan Municipalities Environmental Protection Expenditures Series (2007:1-2024:2)

Linear and nonlinear (Logarithmic, Quadratic, Exponential, Cubic, Compound, Polynomial) Trend Analyses were performed to obtain models which are suitable for the time series structure. The result of the trend analyses showed that the data has an increasing trend. The F Test applied to examine the validity of the trend analyses, and the t-test applied to examine the statistical significance of the model coefficients were found to be significant at the 0.05 level. Summary information on Trend Analyses is shown in the Table 1.

Equation	Model Summary			Parameter Estimates				
	R	F	Df	Sig.	Constant	b1	b2	b3
Linear	0,389	43,313	1	0,000	-	115306,757		
Logarithmic	0,209	17,918	1	0,000	-	1888209,055		
Quadratic	0,555	41,723	2	0,004	1899195,606	-180269,923	4163,052	
Exponential	0,651	126,819	1	0,000	203604,027	,049		
Cubic	0,647	40,375	3	0,000	-	354689,989	-14540,612	175,621
Compound	0,649	126,819	1	0,000	203604,027	1,050		

Table 1. Model Summary of Trend Analysis

We applied various methods such as multiplicative and additive to determine the seasonal effects and seasonal index values of the data. In calculating the average weights of the movements, since the series shows three-month periodicity, it was calculated in the range of Period+4 (Endpoints Weighted by 0.5) and the seasonal index values obtained as a result of the analysis are shown in the Table 2.

Quarterly Periods	Seasonal Index (%)
January-February-March	30,9
April-May-June	73,4
July- August-September	116,3
October-November-December	179,5

Table 2. Seasonal Index Values of Environmental Protection Expenditures Series

In Table 2, it is seen that the data reached the lowest value in the first three months of the year (January-February-March); it is seen that it reaches the highest value in the last three months of the year (October-November-December).

#### 2.2. Exponential Smoothing Method

The Exponential Smoothing method is widely used forecasting approach that applies exponentially decreasing weights to past observations. This ensures that recent data points carry more weight in the forecast than older ones (Holt, 2004: 5). In this method, the weighted average of the values of the past period is calculated and this value is taken as the forecast value of the future period. The weight values tend to decrease exponentially towards the past period. Although the weights tend to decrease rapidly as one goes to the past periods, they never become zero. The Seasonal Exponential Smoothing method is recommended if the analysed series consists of both trend and seasonal (monthly, quarterly) data. In this method, a three-step smoothing is applied to estimate the mean value, trend component, and season component of the series. Each process has a separate exponential smoothing constant. The most important advantage of this method is expressed as being able to reflect trends and seasonal movements (Armutlulu, 2008). Holt-Winters' method, which incorporates Winters' seasonal component into the double-parameter (level, trend) exponential smoothing algorithm developed by Holt (1960), is based on three correction equations-one for the level, one for the trend, and one for the seasonality. We use these fundamental equations to model the seasonal component additively in the Holt-Winters method:

$Lt = \alpha(Yt - St - s) + (1 - \alpha)(Lt - 1 + bt - 1)$	(1)
$ht = \beta(It - It - 1) + (1 - \beta)ht - 1$	(2)

Dl = p(Ll)	$L\iota$ I) $(I$		(2)
$St = \gamma(Yt)$	$-Lt) + (1 - \gamma)$	)St-s	(3)

Ft+m = Lt + btm + St-s+m	(4)

where S; length of seasonality (for example, number of months or quarters), *Lt*; the level of the series, *bt*; trend component, *St*; seasonal component and *Ft+m*; m period future forecast value (Makridakis et al., 1997). Due to the trend and seasonal components in the time series examined in the study, it can be stated that the method that can best reveal the properties of the series among the exponential smoothing methods is the Seasonal Exponential Smoothing (Holt-Winter's) method. We tested various models by applying transformations such as the square root and natural logarithm to the series. Among the models we tested, we found that the most suitable

exponential smoothing model was the 'Holt-Winter's Multiplicative Seasonal' model for the series without transformation. Smoothing constants (parameter estimates) and model summary of the developed model are given in Table 3.

Model		Estimate	S.E.	t	Sig.
Madal 1	(Level) α	,597	,087	6,844	,000
No Transformation	(Trend) $\beta$	,597	,114	3,617	,001
No Transformation	(Season) $\gamma$	,999	,339	2,945	,004

Table 3. Exponential Smoothing (Seasonal Multiplicative Holt-Winter's) Model Summary

#### 2.3. Box-Jenkins (ARIMA) Method

The ARIMA (Auto Regressive Integrated Moving Average) method, the aim is to determine the best model among the alternative models and to make estimate for the future with the help of this model. The estimation of the variable examined in this method; lagged values of the variable itself, error terms or a combination of both (Bozkurt, 2007). It is stated that the present value of the observed variable in the ARIMA method consists of the weighted sum of the past observation values and the combination of random shocks (Akgül, 2003: 35).

ARIMA (p,d,q) process is stated as;

$$w_t = \phi_1 w_{t-1} + \phi_2 w_{t-2} + \dots + \phi_p w_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q}$$
(5)

where p; the degree of autoregressive process, which is the number of past observation values in the series,  $\phi$ ; coefficient showing the relationship between past observation values and present observation values,  $\theta$ ; weights indicating the parameters that can take negative or positive values,  $\varepsilon_t$ ; error term in the model,  $w_t$ ; refers to the new series in which the  $y_t$  series is made stationary by taking the difference of d. (Akgül, 2003: 105-106). The model, which is generally shown as ARIMA (p,d,q), is expressed as SARIMA (P, D, Q) or ARIMA (P,D,Q)s by including seasonality in the model. The number of seasonal AR and seasonal MA terms are denoted by P and Q, respectively. Where P is the number of seasonal autoregressive components, Q is the number of seasonal moving average terms and D shows the number of seasonal differences required to ensure stability (Meyler et al., 1998: 40). In non-stationary series with seasonality, stationarity should be ensured as in non-seasonal series. For this purpose, the process of taking the difference in monthly series showing seasonal characteristics;

$$\Delta_{12} y_t = y_t - y_{t-12} \tag{6}$$

In quarterly data;

$$\Delta_4 y_t = y_t - y_{t-4} \tag{7}$$

The seasonal ARIMA model for the non-stationary  $y_t$  series,

$$\Delta_{S}^{D} = \frac{\theta(B^{S})}{\phi(B^{S})} \varepsilon_{t} \tag{8}$$

where *s*; seasons, *D*; number of differences,  $\Delta_s^D$ ; the series converted to stationary state after the difference process,  $\emptyset(B^s)$  and  $\theta(B^s)$ ; polynomials of the  $B^s$  at the P and Q orders (Akgül, 2003: 200).

The ARIMA model-building process includes four steps: determining the provisional model structure, estimating parameters, conducting model adequacy tests, and forecasting (Chen et al., 2008: 627). The stationarity of the time series is first evaluated using the autocorrelation function (ACF) and partial autocorrelation function (PACF) (Akgül, 2003). For testing stationarity, the

Augmented Dickey-Fuller (ADF) test, an extension of the unit root test, is commonly employed. If no unit root is detected, the series is stationary, otherwise, it is non-stationary (Li et al., 2017: 16327). In the ARIMA modelling process, once the model structure is determined, the parameters of the provisional model are estimated, ensuring the significance of the coefficients. Parameter significance is tested using t-statistics and p-values, with the p-value typically set at 0.05. If the pvalue is below this threshold, the null hypothesis (that the parameter is zero) is rejected, indicating the parameter is statistically significant (Chen et al., 2008: 629). After estimating the provisional model, its adequacy is evaluated through diagnostic checks (Wang and Meng, 2012: 1186). In applying the Box-Jenkins (ARIMA) method, we first performed stationarity analyses on the data. As shown in Figure 1, we observed that the series lacked stationarity in both mean and variance, and the seasonal component was not constant over time. To address the variability in the seasonal component, we applied a natural logarithm transformation to the series. We used the Augmented Dickey-Fuller (ADF) test to assess non-seasonal stationarity in the logtransformed series, and we employed the HEGY (Hylleberg, Engle, Granger, and Yoo) test to evaluate seasonal stationarity. The results of these stationarity analyses indicated that the series achieved stationarity by taking the first-degree non-seasonal difference and the second-degree seasonal difference. Therefore, the appropriate degrees of differencing were determined to be d=1 for the non-seasonal difference and D=1 (S=4) for the seasonal difference.

By examining the autocorrelation (ACF) and partial autocorrelation (PACF) functions of the data, it has been determined that the appropriate Box-Jenkins model for the metropolitan municipalities environmental protection expenditure series, whose first degree non-seasonal difference and first degree seasonal difference is taken, is the SARIMA  $(0,1,0)(0,1,0)_4$  model, which is expressed as the "Multiplicative-Seasonal ARIMA Model". The final parameter estimates and model summary for the determined model are shown in the Table 4. As can be seen in the table, all the t-values related to the parameter estimates of the determined SARIMA model are statistically significant at the 0.05 significance level. In addition, the critical value for the t-test statistic used in testing the statistical significance of seasonal lags in the Box-Jenkins method is taken as 1.25. The seasonal parameter estimates were found to be significant because they were |t| > 1.25.

Process	Estimate	Standard Error	t-Statistics	p-value	
Outlier-Q1 2012 (Additive)	0,429	0,072	5,928	0,000	
Outlier-Q1 2018 (Additive)	0,342	0,071	4,731	0,000	
Number of Observations	6	70			
Number of Observations After Differences		65			
R <sup>2</sup>		0,980			
MAPE		11.053			
<b>Bayesian Information Cr</b>	iterion (BIC)	26,550			
Transformation		Natural Logarithm			
Difference		Nonseasonal (1) difference and seasonal differences (2)			

**Table 4.** SARIMA (0,1,0)(0,1,0)<sub>4</sub> Model Summary

Ljung and Box  $(Q^*)$  test statistics were used to check whether the residuals of the model were random (white noise) and whether they were correlated after determining the model structure and estimating the model parameters. For seasonal models;

$$Q^* = n(n+2) \sum_{k=0}^{\infty} \frac{r_k^2}{n-k} \sim \chi^2 (k-p-q-P-Q)$$
(9)

 $Q^*$  statistic obtained by the equation above has  $\chi^2$  distribution at (*k*-*p*-*q*-*P*-*Q*) degrees of freedom under the hypothesis " $H_0$ :  $r_1 = r_2 = \cdots r_k = 0$ ". In the calculations made for the 4th, 8th and 12th lags of the residual series of the model, it was seen that  $Q^* < \chi^2$  was found, and therefore the  $H_0$  hypothesis was accepted at the 0.05 significance level. The results of the applied Ljung-Box test are given in Table 5.

**Table 5.** Ljung-Box Test Summary

Ljung-Box (Q)				
Statistics	DF	Sig.		
15,572	18	0,622		

This confirms that there is no significant relationship between the errors, that the errors are randomly distributed, in other words, it has a white noise process, and therefore the determined SARIMA  $(0,1,0,)(0,1,0)_4$  model is appropriate. The "IBM SPSS 23.0" statistical package program was used in modelling the data with the Box-Jenkins method and in obtaining the autocorrelation (ACF) and partial autocorrelation (PACF) functions of the models evaluated in the estimation phase.

#### 2.4. Artificial Neural Networks (ANN) Method

Economic time series are affected by many political, international and natural shocks. This means that the irregular component will remain significant in economic time series, and it will be extremely difficult to forecast (Hansen and Nelson, 1997). Due to the difficulties of the estimation of economic data, which does not always exhibit a linear feature by nature, it can usually be done with linear methods. However, this may lead to incorrect results if the analysis includes possible nonlinear structures (Yurtoğlu, 2005). With their ability to model nonlinear patterns and learn from data, ANNs provide an important forecasting tool to identify trend cycle and seasonal components (Hansen and Nelson, 1997: 872). An important advantage of ANN models over other nonlinear model classes is that they produce predictions with a high degree of accuracy for a very large data set. In addition, the parallel processing of the information in the data, the fact that it does not need a previously accepted model form, prior knowledge and assumptions in the model creation process are expressed as the superior aspects of ANNs. In the ANN method, the model is mostly determined by the characteristics of the data (Zhang, 2003: 163). Developed with inspiration from a real brain structure, ANNs consist of the interconnections of elements expressed as neurons or processors. Each neuron in the network processes the inputs connected to it and generates an output signal (Svozil et al., 1997: 44). An artificial neuron consists of inputs, weights; sum up function, transfer function and output. The Figure 2 shows the basic elements of an artificial neuron (Beckenkamp, 2002: 26).



Figure 2. Basic Elements of Artificial Neuron Source: Beckenkamp, 2002

The artificial neuron has several inputs  $(x_j)$  and one output  $(y_j)$ . Each input link has an associated weight, usually a real number, that controls the link strength  $(w_{ij})$ . Weights can be inhibitory or stimulating (typically negative and positive values). The net input is calculated by multiplying the input values by the corresponding weights and adding the threshold value  $(b_j)$ . The output value is calculated by applying an activation function using  $(net_i)$ . The first equation below shows the net input, the second equation shows the output value (Beckenkamp, 2002: 27):

$$net_i = \sum_j x_j w_{ij} + b_j \tag{10}$$

$$y_i = f_i(net_i) \tag{11}$$

Inputs;  $(x_1, x_2, x_3 \dots x_i)$  is the information coming to the artificial neuron from the outside world, other neurons or the neuron itself. Inputs (determined by the samples that ANN is required to learn (Oztemel, 2006). Weights;  $(w_1, w_2, w_3, ..., w_i)$  are the coefficients that show the effect and importance of the inputs to the artificial nerve cell (neuron) on the cell. Each input has its own weight. The weight value indicates that the input data has a strong connection with the neuron or that it is important for the neuron (Elmas, 2011). Summing (Combination) Function; It is the function in which the net input to the artificial neuron is calculated. Here, each incoming input value is multiplied by its own weight and summed, so that the net input is calculated. Activation Function: It is the function in which the neuron output is obtained by processing the net information obtained from the summing function (Öztemel, 2006). The activation function, also referred to as the transfer function, determines the relationship between the neuron, the inputs of the network and the outputs of the network, it considers the nonlinearity, which is important for ANN application (Zhang et al., 1998: 46-47). In models related to ANN, different activation functions are selected according to the work done by the network. Frequently used activation functions are TanSig (tangent sigmoid), LogSig (logarithmic sigmoid) and Purelin (Linear) functions. The TanSig activation function shows a non-linear change in the [-1,1] range. The LogSig activation function shows a non-linear change in the [0,1] range. Purelin activation function shows a linear change (Altun et al., 2002). In practice, sigmoid function and hyperbolic tangent (tangent sigmoid) function are preferred (Yavuz and Deveci, 2012). Output; is the output value obtained from the activation function. This output value obtained can be sent to the outside or to another neuron (Öztemel, 2012). The result of the summing function is passed through the activation function and sent to the output. It does not produce value output below the threshold level of the neuron transfer function. For the neuron to produce an output, it must have a value above the threshold value in the activation function (Elmas, 2011: 32). In the ANN method, models based on the prediction of future periods based on past observation data are expressed

as NAR (Nonlinear Autoregressive) artificial neural network models. In the NAR model, the future value  $(y_t)$  of the series is output as the input data of the past observation data  $(y_{(t-1)}, y_{(t-2)}, \dots, y_{(t-p)})$ . The model is shown mathematically as follows.

$$y_t = f(y_{(t-1)}, y_{(t-2)}, \dots, y_{(t-p)}, w) + \varepsilon_t$$
(12)

Where p is the number of feedback delays (indicated as 1:4 for p=4), w is a vector of all parameters and f is a function determined by the network structure and link weights. Therefore, the neural network is equivalent to a nonlinear autoregressive model (Zhang, 2003: 163). The Nonlinear Autoregressive Artificial Neural Network (NAR) network structure is given in Figure 3.



**Figure 3.** Nonlinear Autoregressive Artificial Neural Network (NAR) **Source:** Ruiz et al., 2016.

NAR network is known as a powerful model class in the estimation of nonlinear time series, thanks to dynamic repetitive network structures consisting of several layers and feedback connections between ANNs. In the NAR model, the future values depend only on the previous values (p) of the output signal. A closed-loop NAR network is used to perform a multi-step forecast. In the given equation below, the output of the closed-loop NAR network is expressed as follows to show the future prediction steps "d" (Wang et al., 2017; Benmouiza and Cheknane, 2016).

$$y_{(t+d)} = f(y_{(t-1)} + y_{(t-2)} + \dots + y_{(t-p)}, w) + \varepsilon_t$$
(13)

The modelling process in ANN is briefly; it consists of determining the appropriate topology, training phase, testing and forecast phases. Determining the topology, which is the first step of the modelling phase, has a key role in solving the problem (Svozil et al., 1997). At this stage, it is necessary to determine the number of hidden layers and the number of neurons in the network in the creation of the NAR artificial neural network, which will be used to forecast. Although different algorithms have been proposed at this stage, the "trial and error" method is commonly used (Wei et al., 2012). Increasing the number of parameters excessively in the creation of the network structure causes the system to become more complex; not using enough parameters can cause the generalization feature of the network to be negatively affected (Ruhan and Chen, 2018: 2672). Before starting the training phase in artificial neural networks, the data must be normalized and the activation function must be determined (Hamzacebi, 2011). The next stage is training phase after determining the appropriate architectural structure in ANNs. During the training phase, the connection weights between the neurons in the layers of the network are updated. The weight values determined randomly in the first stage are updated according to the relationship between the input and output values provided to the ANN, until the ideal result is reached. The update process may differ according to the selected learning algorithms. The learning algorithms chosen according to the nature of the problem may also vary. The next stage is the testing stage after the training process is completed in ANNs. In the test phase, input data that it has not seen

before is given to the network and in response to these data, the network is provided to produce output. At this phase, no change is made regarding the network weights, and the output values produced by the network are compared with the actual values. It is checked whether the outputs produced by the ANN adequately overlap with the actual output values (Öztemel, 2006). The forecast phase is started after the completion of the training and testing phase. The ANN model to be used in the predictions is determined by considering of the forecasting performances of the experimental models. In this study, we used the MAPE criterion to evaluate the prediction performance of the models. We developed a "Non-Linear Autoregressive (NAR) ANN" model to forecast expenditures, utilizing the environmental protection expenditure data of the Metropolitan Municipalities for the period 2007:1 to 2024:2. In the first stage of the model creation process, the data used in the application were normalized in the range of [-1 +1]. The following equation in the deep learning toolbox (MATLAB. Deep Learning Toolbox<sup>TM</sup>. Reference. R2022b) was applied for the normalization process.

$$y = (ymax - ymin) * \frac{x - xmin}{xmax - xmin} + ymin$$
(14)

where y is the normalized value, *xmax* is the largest value in the data set, *xmin* indicates the smallest value in the data set and x is the data value to normalize. Since the data will be normalized in the range of [-1 +1], *ymin* is selected as -1 and *ymax* as +1. MSE (Mean Squared Error) statistics were used to measure the training performance of the network, and MAPE (mean absolute percent error) statistics were used to measure model performance. As the activation function, tansig (tangent-sigmoid) is used in the hidden layer. "Bayesian Regularization" and "Levenberg–Marquardt" algorithms, which are frequently used in practice, were chosen as training algorithms. "Bayesian Regularization" is a training algorithm that needs only training and test data as a dataset. "Levenberg–Marquardt", on the other hand, needs validation data in addition to training and test data. For this reason, 80% of the data set was used for training and 20% for testing the network for models using "Bayesian Regularization" as the training algorithm. For models using Levenberg-Marquardt as the training algorithm, 70% of the data set was used for training, 15% for testing and 15% for validation of the network.



Figure 4. NAR Network with [1:6:1] Architecture Used in Practice (Open Loop)

Network structures with different architectures, with a delay vector consisting of one hidden layer and one output layer, ranging from 1:1 to 1:20, and the number of neurons in the hidden layer ranging from 1 to 15, were tested using the MATLAB R2022b Deep Learning toolbox. In order to determine the model with the highest prediction performance, the NAR model with 15 delays, with one hidden layer (6 neurons), which has the highest prediction accuracy according to the MAPE statistics, was selected for use to forecast. The Nonlinear Autoregressive (NAR) ANN model chosen to be used in the application is shown in Figure 4. The training phase of the model was started after the architectural structure of the NAR model chosen to be used in the

application was determined. At this stage, it is possible to observe the training and test performance of the model. Figure 5 shows the training and test performances of the model.



Figure 5. Training and Test Performance of NAR Network with [1:6:1] Architecture

In Figure 5, it is seen that at the end of 150 iterations, the network reduces the mean square error to 8.1079e-05 by using the data allocated for the training set. It is also seen in that the training performance of the network gives better results than the test performance.



Figure 6. Regression of NAR Network with [1:6:1] Architecture

In the analysis, it was observed that as the number of neurons increased, the training success of the network increased, but the test success decreased. This means that the network produces successful results using the data it has seen before but cannot produce successful results using the data it has not seen before. The results of the regression analysis regarding the training and test data are included in Figure 6. It is seen that the R value is calculated as 0.99973 when the regression analysis of the NAR network for all the data is examined.

Figure 7 shows the autocorrelation revealing the relationship between the forecasting errors that occur during the training phase of the network.



Figure 7. Error Autocorrelation of NAR Network with [1:6:1] Architecture

In the guide for the deep learning toolbox, it is stated that the error autocorrelation function shows the relationship of the estimation errors with time, and in a perfect estimation model, there is only one non-zero value in the autocorrelation function, and this value occurs at the zeroth lag, and this value shows the mean squared error. In other words, this means that there is no relationship between the estimation errors. It is stated that the values other than the zero lag of the created model are within the 95% confidence interval, indicating the adequacy of the model (Beale et al., 2019: 117). The network was turned into a closed loop to be used to forecast after the completion of the training and testing phases for the created NAR network.

# 2.5. Empirical Findings and the Environmental Protection Expenditure Estimates of Metropolitan Municipalities

The results of the forecasting performances of various models created using different methods to be used in the forecasting of the environmental protection expenditures of metropolitan municipalities are shown in the Table 6.

1	0			
Method	Model	MAPE (%)		
Exponential Smoothing Method	Holt-Winter's Multiplicative Seasonal	11,931		
Box-Jenkins Method	SARIMA (0,1,0,)(3,2,0)4	11,03		
Artificial Neural Networks Method	The Nonlinear Autoregressive (NAR)	8 90		
Artificial Neural Networks Method	Network	0,90		

Table 6. Comparison of Forecast Performances of Models Created using Different Methods

Models with a Mean Absolute Percentage Error (MAPE) value of less than 10 are considered very accurate, while those with MAPE values between 10-20 are considered good models. Models with MAPE values between 20-50 are deemed acceptable, and those with MAPE values above 50 are considered faulty (Witt and Witt, 1992: 137; Lewis, 1982: 40). Upon examining the MAPE results in Table 4, which show the forecast performances of the models used to predict the environmental protection expenditures of metropolitan municipalities, it is evident that the forecasts made by

the Seasonal Exponential Smoothing method, the Box-Jenkins method, and the Artificial Neural Networks (ANN) method all provided good results. Although the NAR [1:6:1] ANN Model yielded the lowest (8,90) MAPE rate, we generated the environmental protection expenditures for the periods 2024:1 to 2025:4 using all three models of methods, as they showed the best prediction performance among their counterparts. We present the forecast values generated by the developed methods/models in Table 7.

Periods	Artificial Neural Networks (NAR)	Box-Jenkins (SARIMA)	Seasonal Exponential Smoothing Method (Holt-Winter's)
2024: 1	7087712,542	6977340	7185636
2024: 2	16101553,89	16259159	16009546
2024:3	23241539,67	24073050	23479397
2024: 4	34732746,7	35131637	33974306
2025: 1	15507204,83	15647589	15773437
2025: 2	20402112,37	21178661	21195992
2025: 3	24009990,92	23040796	24008227
2025: 4	32415304,98	33109289,7	33237987

Table 7. Forecast Values Generated by the Methods Used in Study (Thousand TL)

## 3. CONCLUSIONS

This study investigated the forecasting performance of three different time series modelling methods-Exponential Smoothing (Holt-Winters), Box-Jenkins (ARIMA) and Artificial Neural Networks (ANN)-in predicting environmental protection expenditures of metropolitan municipalities in Turkey. Our primary objective was to identify the method with the highest prediction accuracy to assist policy makers and local governments in future budget planning for environmental protection services. The results show that while all three methods are capable of producing reliable forecasts, the ANN model consistently produced the highest accuracy, followed closely by Holt-Winters and SARIMA. We evaluated these models using the Mean Absolute Percentage Error (MAPE), which confirmed the superior predictive power of the ANN model for this dataset. However, predictions from all three models were generated to ensure robustness and provide a broader comparison of forecasting approaches. The results are particularly relevant for local governments and policy makers, as accurate forecasting of environmental protection expenditure is crucial for the strategic allocation of financial resources. This is becoming increasingly important as environmental protection activities often require significant investments, especially in environmentally friendly technologies, which are key to a sustainable development approach in Turkey. Failure to accurately forecast these expenditures can lead to resource shortages, disruptions to planned activities and projects, and financial imbalances. Therefore, the results of the study are significant in determining the resource requirements for environmental protection activities, exploring alternative financing options, and strengthening the relationship between budget planning and project implementation.

#### 3.1 Theoretical Implications

The application of Exponential Smoothing, Box-Jenkins (ARIMA), and Artificial Neural Networks (ANN) in this study contributes to the theoretical literature on forecasting methodologies, particularly within the context of environmental budgeting. While forecasting methods are commonly used in financial data and public expenditure estimation, their application to environmental protection expenditures has been limited. By integrating both

traditional and machine learning-based methods, this study highlights the potential of these models in addressing the nonlinearities and complexities inherent in environmental protection expenditure data, which are often influenced by predictable trends and irregular external shocks. The study also offers theoretical insights into the growing importance of artificial intelligence in public finance, as ANN's ability to capture complex, nonlinear patterns in data presents an alternative to conventional linear models. The superior predictive accuracy of ANN in long-term forecasting reinforces the potential for machine learning techniques to complement or even outperform traditional time series models in contexts characterized by nonlinear relationships. This supports the broader application of machine learning in public finance research, especially where dynamic and complex data patterns exist. Furthermore, the comparative analysis of model performance enhances our understanding of forecasting methods in public expenditure management. Although all three methods demonstrated acceptable levels of accuracy, ANN consistently delivered more precise results. This finding aligns with existing literature on machine learning models' ability to capture intricate patterns, suggesting that ANN and similar techniques should be more widely applied in public finance research, especially in forecasting complex datasets like environmental protection expenditures.

# 3.2. Managerial and Practical Implications

From a managerial and practical standpoint, the findings of this study hold significant implications for local governments and policymakers in Türkiye. First, the study demonstrates the utility of advanced time series forecasting models in predicting future environmental protection expenditures, which is vital for efficient budget allocation and long-term planning. The accurate forecasting of these expenditures can help local governments better manage resources, prevent budgetary shortfalls, and ensure the successful execution of environmental protection projects. The superior performance of ANN in capturing nonlinear and complex relationships suggests that local governments should consider integrating machine learning techniques into their forecasting processes. This can enhance decision-making, allowing policymakers to anticipate future expenditure needs with greater precision. Additionally, the study emphasizes the importance of accounting for seasonal variations in expenditure patterns, which can further improve forecast accuracy and support timely allocation of funds. On a broader level, the study underscores the need for capacity building within public institutions to incorporate advanced forecasting techniques into routine financial planning. Policymakers should invest in training and infrastructure that enable the use of artificial intelligence and machine learning in public finance, thus better preparing local governments to meet future environmental challenges.

# 3.3. Limitations and Directions for Future Studies

While this study provides valuable insights, it is not without its limitations, which present opportunities for future research. Firstly, the dataset used is limited to environmental protection expenditures of Turkish metropolitan municipalities from 2007 to 2022. This relatively narrow timeframe may not fully capture broader trends or external factors, such as economic crises, policy changes, or technological advancements, which could influence future expenditures. Future studies could extend the dataset to cover a wider range of years and include diverse macroeconomic conditions for a more robust analysis. Secondly, the study focuses on three wellestablished forecasting methods—Exponential Smoothing, ARIMA, and ANN. While these methods are effective, future research could explore the application of more advanced models, such as Support Vector Machines (SVM), Long Short-Term Memory (LSTM) networks, or hybrid models that combine different techniques. Incorporating these models could provide a deeper understanding of which methods are most suitable for forecasting public expenditures.

Additionally, this study does not integrate exogenous variables into the forecasting process, relying solely on historical expenditure data. Future research should consider external factors like population growth, inflation, changes in environmental policy, and economic variables that could impact future expenditures. Including these variables would enhance the relevance and accuracy of the forecasts. Moreover, future studies could explore various artificial neural network architectures (e.g., Radial Basis Function Networks, Multi-Layer Perceptron, Time Delayed ANN) and other artificial intelligence methods (e.g., fuzzy logic, genetic algorithms, ANFIS, or SVR) in a comparative manner. This would provide a broader perspective on the potential of advanced models in public finance forecasting. Finally, future research should account for unforeseen events, such as natural disasters or global pandemics, which could drastically alter expenditure patterns. Incorporating stochastic elements or scenario-based forecasting models would allow for a more comprehensive approach to forecasting in public finance.

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